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Original Article

Numerical simulation of time-fractional Navier-Stokes equation in cylindrical coordinates for an unsteady one-dimensional motion of a viscous fluid flow in a given tube

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ABSTRACT

This paper proposed and applied a three-step computational algorithm to solve the time-fractional Navier-Stokes equation (FNS) in a given cylindrical coordinates for one-way unstable flow motion. The Caputo definition of fraction order was obtained using the Riemann Liouville fractional integral operator, which was coded with the MAPLE18 software command and applied to simulate the different fractional values presented in 2D and 3D surface graphs for understanding better the operation of fractional Navier-Stokes equations over time in cylindrical coordinates. We considered different test cases to show the proposed algorithm's efficiency, robustness, and feasibility, which ultimately reduces the computational time and ease of implementation for the simulation of the fractional order of the fractional Navier-Stokes equation considered.

1. Introduction

Fractional partial differential equations are most used in thermodynamics sciences such as reactive diffusion, anomalous diffusion, diffusion-related to gas emission in transportation system, physics of polymers, electrical networks, electrochemical corrosion, chemical sciences, seismic waves propagation, porous structures in thermodynamics, kinetic processes in applied physics, and other similar problems in the study of applied mathematics and engineering [1–7]. The study of the Navier-Stokes equation (NSE) are consider to be one of fundamental models in fluid mechanics which describes the motion of Newtonian fluid flows under two significant conditions: laminar or turbulent flow phenomenon [8, 9]. In the last two decades, the study of time fractional-order Navier-Stokes equation (FNS) have received great attention in studying of fluid mechanics which have proved to be significant in modeling many problems and natural phenomenon in applied mathematics and engineering sciences, such as computer science, inventory science, chemical science, biological sciences, and so on. Several Authors have proposed and applied numerous methods to solve and obtain analytical solutions fractional Navier-Stokes [FNS] equation such as authors [10] employed Adomian decomposition method to solve a time-fractional Navier-Stokes equations, [11] applied homotopy analysis technique for the numerical solution of time-fractional Navier-Stokes equation, [12] Laplace decomposition method was used to obtain analytical solution of fractional

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Navier–Stokes equation, [13] presented numerical techniques for the generalized Navier–Stokes equations, [14] homotopy perturbation technique was presented to solve time-fractional Navier-Stokes equation in a given polar coordinates, [15] proposed FRDTM for the numerical solution of the multidimensional time-fractional model of Navier-Stokes equation, [16] proposed a good analytical technique for solving time fractional Navier-stokes equation, [17] obtained numerical solutions of Navier-Stokes equations with time fractional derivatives using analysis approach, [18] studied the fractional model of Navier-Stokes equation arising in unsteady flow of a viscous fluid, [19] presented numerical solution for the time-fractional Navier-Stokes equations by homotopy perturbation Elzaki transform, [20] presented a new efficient technique for solving fractional coupled Navier-Stokes equations using q homotopy analysis transform method, [21] mild solutions was obtained for the Navier–Stokes equations with a time-fractional derivative and [22] investigated the existence and uniqueness of some properties about the solutions obtained such as, regularity in Lebesgue spaces and decay rate respectively.

2. Navier-Stokes Model

The governing equation for Navier-stokes of the fluid dynamics describes the relationship between velocity, kinematics viscosity, density, pressure, and time of the fluid mechanics. The time-fractional Navier-Stokes equation (NSE) of the form: [23].

$$\begin{cases} D_t^{\alpha}\underline{\Phi} + \left(\underline{\Phi}\cdot\nabla\right)\underline{\Phi} = -\frac{1}{\rho}p\nabla + \upsilon\nabla^2\,\underline{\Phi} \\ 0 < \alpha \leq 1 \end{cases} \tag{1}$$

$$\nabla\cdot\underline{\Phi} = 0 \tag{2}$$

Where $\upsilon, \Phi, \rho, p, t$ are velocity, kinematics viscosity, density, pressure, and time respectively, and α is the fractional-order derivative.

In this paper, we consider equation (1) in cylindrical coordinates for an unsteady one-dimensional motion of a viscous fluid flow which transformed into a cylindrical time-fractional Navier-stokes equation of the form:

$$D_t^{\alpha}\underline{\Phi} = P + \upsilon\left(\frac{\partial^2\Phi}{\partial r^2} + \frac{1}{r}\frac{\partial\Phi}{\partial r}\right) \tag{3}$$

subject to initial condition:

$$\Phi(\mathrm{r}, 0) = G(r) \tag{4}$$

The unsteady flow of a viscous fluid in a given tube where velocity field and one space coordinate are the function and time as a dependent variable are considered. Some authors have worked and obtained numerical solutions of the time-fractional Navier-Stokes equation such as [24] presented and applied transform methods for the analytical solutions for the time-fractional Navier-Stokes equation, [25] studied and obtained numerical solution for fractional-order Navier-Stokes equation of the unsteady viscous flow in thermodynamics study, [26] modified Laplace decomposition method authors [27] applied Adomian decomposition method for the analytical solutions of time-fractional Navier-Stokes equations.

The outline of this paper goes thus, first, we briefly discussed the basics of the fractional partial differential equations and fractional Navier-Stokes equations over a given time. Second, we stated the cylindrical coordinates for an unsteady one-dimensional motion of a viscous fluid flow which transformed the cylindrical time-fractional Navier-stoke equation presented. Third, some basic concepts of fractional calculus that will be useful in this article are identified. Fourth, descriptions and formulation of a three-step computational algorithm are presented and the application of the proposed algorithm for different valuesα. Lastly, we presented the solutions obtained, plots profiles, discussion, and conclusion respectively.

3. Methodology

3.1 The Riemann-Liouville Integral

The Riemann-Liouville integral operator of the fractional-order α of a function $\psi(x,t) \in C_{\mu}, \mu \geq -1$ is of the form:

$$J_t^{\alpha}\Phi(x,t) = \frac{1}{\mathrm{GAMMA}(\alpha)}\int_0^x (t - s)^{\alpha-1}\Phi(x,\tau)\,d\tau, \tag{5}$$

For $\tau > 0$ and $\alpha > 0$

3.2 Description of a three-step computational algorithm

To simulate numerical solutions for equation (3), we develop a three-step algorithm using Riemann-Liouville integral of fractional-order by the MAPLE 18 software coded commands for the computational assessment and behavior pattern of fractional-order in cylindrical time-fractional Navier-stoke equation. The three-step by algorithm goes thus:

Restart:
Step 1
$with(PDEtools)$:
$with(linalg)$:
$withplots$:
$Digits := \mathbb{R}^+$;
$N := \mathbb{R}^+$;
$P := \mathbb{R}^+$;

$\upsilon := \mathbb{R}^+;$
$\Phi(r,0) := G(r);$
$\Phi[0] := \left(\Phi(r,0)\right);$

Step 2:

for n from 0 to N do

$$\Phi^* := P + \upsilon * \left(diff(\Phi[n], r\$2) + \frac{1}{r} * diff(\Phi[n], r)\right);$$

$$\Phi[n+1] := simplify\left(\frac{1}{\mathrm{GAMMA}(\alpha)} * int((r-s)^{\alpha-1} * subx(s=r, \Phi^*), s = 0 \ldots r), assume = nonnegative\right);$$

end do;

Step 3:

$\mathrm{solFNSE} := \mathrm{sum}(\Phi[\mathrm{k}], \mathrm{k} = 0 \ldots \mathrm{N}+1);$
$\alpha_1 := [0.2, 0.4, 0.6, 0.8, 1.0]:$
$\alpha_2 := [0.02, 0.04, 0.06, 0.08, 0.1]:$
$\alpha_3 := [0.002, 0.004, 0.006, 0.008, 0.01]:$
for i from 0 by 0.02to 1 do
$sol[1] := evalf\left(eval(\Phi[0.2], [r = \mathrm{i}, t = 5.0])\right);$
$sol[2] := evalf\left(eval(\Phi[0.4], [r = \mathrm{i}, t = 5.0])\right);$
$sol[3] := evalf\left(eval(\Phi[0.6], [r = i, t = 5.0])\right);$
$sol[4] := evalf\left(eval(\Phi[0.8], [r = i, t = 5.0])\right);$
$sol[5] := evalf\left(eval(\Phi[1.0], [r = i, t = 5.0])\right);$
end do;
$\Phi[3Dplot] := plot([\Phi[0.2], \Phi[0.4], \Phi[0.6], \Phi[0.8], \Phi[1.0]], r = 0 \ldots 1, t = -\pi \ldots \pi, color[blue, yellow, green, purple, red], grid = [100,100], title = \text{Time} - \text{fractional Navier} - \text{Stokes in cylindrical coordinates } \Phi(r,t)\ );$

$\Phi[2Dplot] := plot([\Phi[0.2], \Phi[0.4], \Phi[0.6], \Phi[0.8], \Phi[1.0]], r = -1..1, color[lue, yellow, green, purple, red], axes = boxed, title = \text{Time} - \text{fractional Navier} - \text{Stokes in cylindrical coordinates } \Phi(r,t) \text{ at} = 5.0);$

Output: See Tables 2, 3, 4, Figures (1(a), 1(b)… 3(a), 3(f)), and Appendix

4. Computational experiments and results

In this section, we carried out a computational assessment of fractional-order appeared in equation (3) for the numerical simulation of the unsteady flow of a viscous fluid in a tube when pressure is greater than given velocity. We hereby apply three-step algorithm discussed in last section and consider the following parameters.

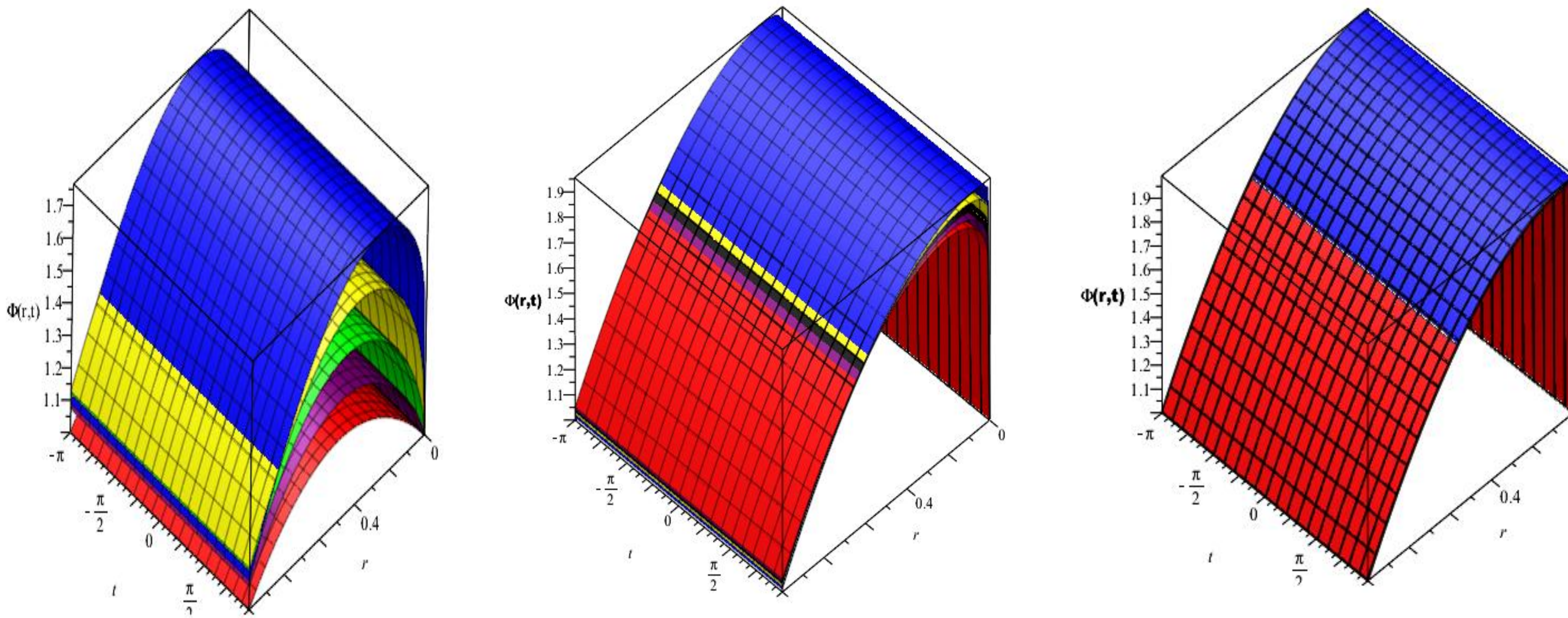
Table 1. Simulation parameters

	Test cases (Decrease in Fractional-order by 100 percent)
α_1	0.2,0.4,0.6,0.8,1.0
α_2	0.02,0.04,0.06,0.08,0.10
α_3	0.002,0.004,0.006,0.008,0.010
P	5.0
υ	1.0

Where P is the pressure and $\boldsymbol{\upsilon}$ is the velocity.

Table 2. Numerical simulation results when $t = 5.0$

$\Phi(r,t)$	α	**Test case 1**	α	**Test case 2**	α	**Test case 3**	α	**Test case 4**	α	**Test case 5**
	0.2	1.00000000	0.4	1.00000000	0.6	1.000000000	0.8	1.00000000	1.0	1.00000000
(0,5)	0.02	1.00000000	0.04	1.00000000	0.06	1.000000000	0.08	1.00000000	0.10	1.00000000
	0.002	1.00000000	0.004	1.0000000	0.006	1.000000000	0.008	1.00000000	0.01	1.00000000
	0.2	1.74937523	0.4	1.55205114	0.6	1.386104361	0.8	1.25627522	1.0	1.16000000
(0.2,5)	0.02	1.93924807	0.04	1.91831392	0.06	1.897244566	0.08	1.87608432	0.10	1.85487489
	0.002	1.95793440	0.004	1.95586654	0.006	1.95379646	0.008	1.95172422	0.01	1.94964986
	0.2	1.74675403	0.4	1.62121617	0.6	1.485853440	0.8	1.35584512	1.0	1.24000000
(0.4,5)	0.02	1.83291787	0.04	1.82525579	0.06	1.817045369	0.08	1.80831765	0.10	1.79910316
	0.002	1.83931879	0.004	1.83863150	0.006	1.837938155	0.008	1.83723878	0.01	1.83653342
	0.2	1.62334914	0.4	1.55877195	0.6	1.463737301	0.8	1.35349730	1.0	1.24000000
(0.6,5)	0.02	1.64100247	0.04	1.64136555	0.06	1.641106327	0.08	1.64024214	0.10	1.63879061
	0.002	1.64012950	0.004	1.64025245	0.006	1.640368885	0.008	1.64047880	0.01	1.64058222
	0.2	1.40158686	0.4	1.39082091	0.6	1.338930759	0.8	1.25813852	1.0	1.16000000
(0.8,5)	0.02	1.36677848	0.04	1.37295510	0.06	1.378536344	0.08	1.38352923	0.10	1.38794132
	0.002	1.36070510	0.004	1.36140413	0.006	1.362097105	0.008	1.36278401	0.01	1.36346486
	0.2	1.08912442	0.4	1.12706049	0.6	1.119174954	0.8	1.07367127	1.0	1.00000000
(1.0,5)	0.02	1.01128165	0.04	1.02203695	0.06	1.032264831	0.08	1.04196480	0.10	1.05113700
	0.002	1.00115180	0.004	1.00229836	0.006	1.003439673	0.008	1.00457572	0.01	1.00570652

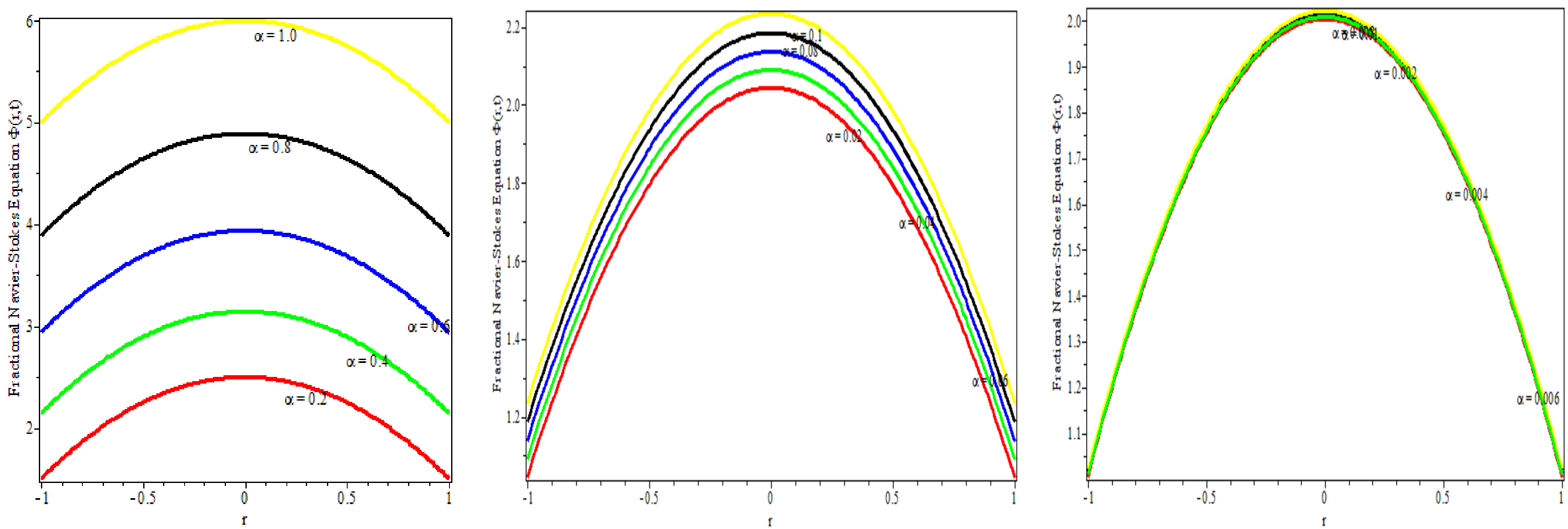


a) for variation of $\alpha = 0.2\ blue,\ \alpha = 0.4\ yellow, \alpha = 0.6\ green, \alpha = 0.8\ purple, \alpha = 1.0\ red$

b) for variation of $\alpha = 0.02\ blue,\ \alpha = 0.04\ yellow, \alpha = 0.06\ green, \alpha = 0.08\ purple, \alpha = 0.1\ red$

c) for variation of $\alpha = 0.002\ blue,\ \alpha = 0.004\ yellow, \alpha = 0.006\ green, \alpha = 0.008\ purple, \alpha = 0.01\ red$

Fig 1. Depicts 3D surface plots for solutions fractional-order in cylindrical time-fractional Navier-stoke equation

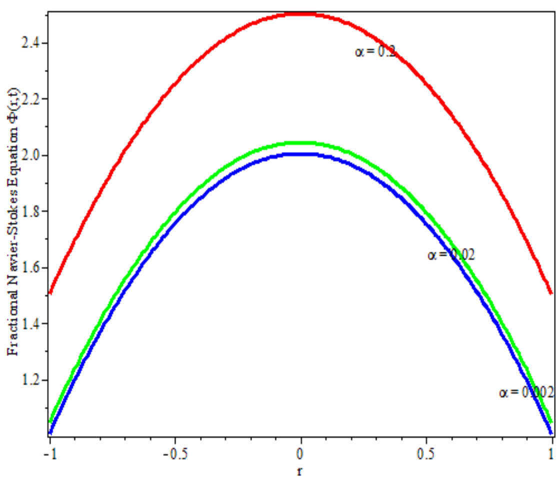


a)
$\alpha = 0.2\ blue, \alpha = 0.4\ yellow,$
$\alpha = 0.6\ green, \alpha = 0.8\ purple, \alpha = 1.0\ red$
$when\ t = 5\ on\ the\ interval\ \ -1 \leq r \leq 1$

b)
$\alpha = 0.02\ blue, \alpha = 0.04\ yellow, \alpha = 0.06\ green,$
$\alpha = 0.08\ purple, \alpha = 0.10\ red$
$when\ t = 5\ on\ the\ interval\ \ -1 \leq r \leq 1$

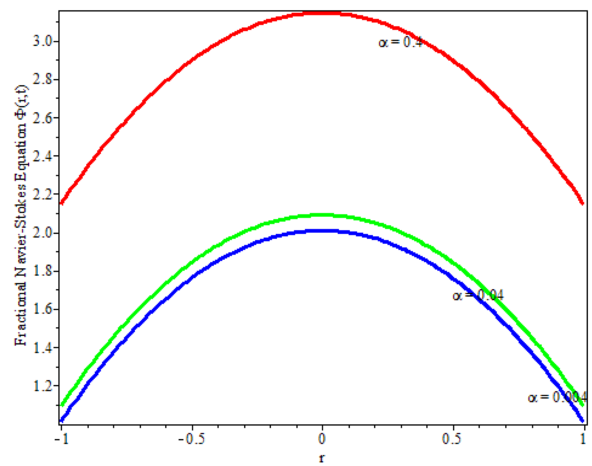
c)
$\alpha = 0.2\ blue\ \alpha = 0.002\ blue, \alpha = 0.004\ yellow,$
$\alpha = 0.006\ green, \alpha = 0.008\ purple, \alpha = 0.010\ red$
$when\ t = 5\ on\ the\ interval\ \ -1 \leq r \leq 1$

Fig 2. Depicts 2D plots solutions for various fractional-order



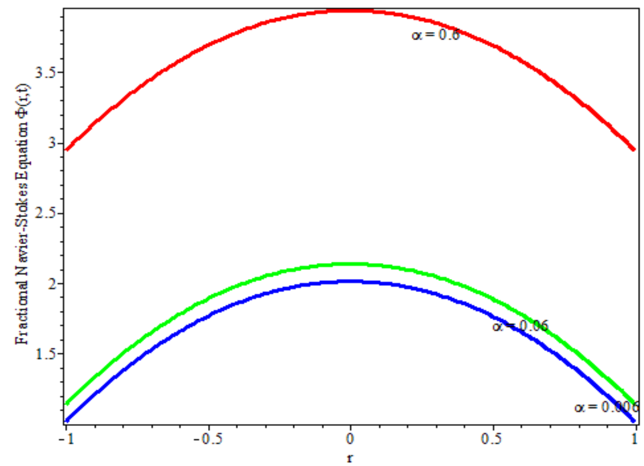
a)

$\alpha = 0.2\ red, \alpha = 0.02\ green, \alpha = 0.002\ blue$
$when\ t = 5, on\ the\ interval\ -1 \le r \le 1$



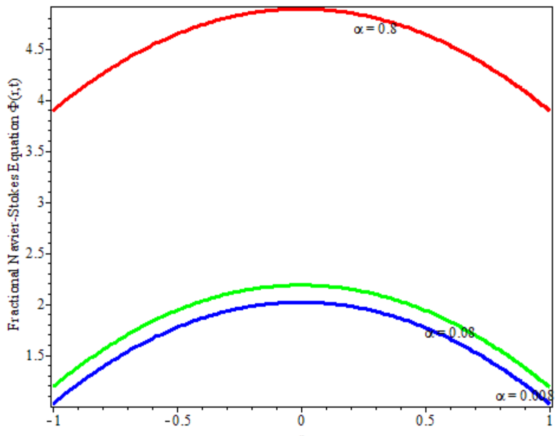
b)

$\alpha = 0.4\ red, \alpha = 0.04\ green, \alpha = 0.004\ blue,$
$when\ t = 5\ on\ the\ interval\ -1 \le r \le 1$



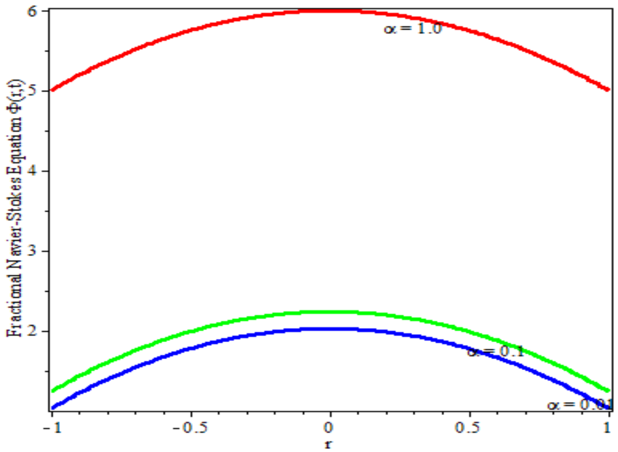
c)

$\alpha = 0.6\ red, \alpha = 0.06\ green, \alpha = 0.006\ blue$
$when\ t = 5, on\ the\ interval\ -1 \le r \le 1$



d)

$\alpha = 0.8\ red, \alpha = 0.08\ green, \alpha = 0.008\ blue$
$, when\ t = 5\ on\ the\ interval\ -1 \le r \le 1$



e)

α=1.0 red α=0.10 $green$,α=0.010 $blue$
$when\ t$=5 $on\ the\ interval\ -1 \leqslant r \leqslant 1$

Fig 3. Depicts 2D plots solutions for various fractional-order

5. Discussion of the Results

The variation of fractional-order α for different values of parameters on time-fractional Navier-Stokes equation (3) are considered for computational simulations when the pressure and velocity are 5 and 1 respectively (see Table 1). We, therefore make the following findings:

1. Tables 2 and 3 show the simulation results obtained at different fractional-order α and observed that increases in the fractional-order α lead to a decrease in the numerical solutions obtained.
2. Fig.1 (a), Fig.1 (b), and Fig.1(c) depicts 3Dplots surface for the simulated results for time-fractional Navier-Stoke equation (1) when fractional-order α decreased by 100 percent respectively.
3. Fig.2 (a), Fig.2 (b), and Fig.2 (c) depicts 2Dplots for the simulated results for time-fractional Navier-Stoke equation (3) when fractional-order α decreased by 100 percent respectively.
4. Fig.3 (a) depicts the decrease in simulation results from fractional-order $\alpha = 0.2$ to $\alpha = 0.002$ and the highest results are recorded in red $\alpha = 0.2$.
5. Fig.3 (b) depicts the decrease in simulation results from fractional-order $\alpha = 0.4$ to $\alpha = 0.004$ and the highest results are recorded in red $\alpha = 0.4$.
6. Fig.3 (c) depicts the decrease in simulation results from fractional-order $\alpha = 0.6$ to $\alpha = 0.006$ and the highest results are recorded in red $\alpha = 0.6$.
7. Fig.3 (d) depicts the decrease in simulation results from fractional-order $\alpha = 0.8$ to $\alpha = 0.008$ and the highest results are recorded in red $\alpha = 0.8$.
8. Fig.3 (e) depicts the decrease in simulation results from fractional-order $\alpha = 1.0$ to $\alpha = 0.01$ and the highest results are recorded in red$\alpha = 1.0$.

Generally, the simulation results in this framework demonstrated the relationship between pressure and velocity when $P > \upsilon$ on the unsteady flow of a viscous fluid in a tube which was described by the time-fractional Navier-Stoke equation

6. Conclusion

In this paper, we proposed a three-step computational algorithm for the numerical solutions of the time-fractional Navier-Stoke equation cylindrical coordinates when pressure required is greater the velocity and the proposed algorithm demonstrated a good computation technique and reliable approach of solving different fractional-order partial differential models arising in fluid dynamics and chemical engineering. From numerical solutions obtained, it is seen that the rate of converges is fast and a good agreement with analytical solutions. Hence, we conclude that the present algorithm is powerful with high reduction in computational lengths and straightforward to obtain analytical-numerical solutions in many areas of time-fractional problems in fluid mechanics.

Appendix

Step 1:

```
restart :
with(PDEtools);
with(plots) :
Digits := 12;
N := 4;
Φ[r, 0] := 1 - r^2;
Φ[0] := Φ[r, 0];
```

[CanonicalCoordinates, ChangeSymmetry, CharacteristicQ, CharacteristicQInvariants, ConservedCurrentTest, ConservedCurrents, ConsistencyTest, D_Dx, DeterminingPDE, Eta_k, Euler, FromJet, FunctionFieldSolutions, InfinitesimalGenerator, Infinitesimals, IntegratingFactorTest, IntegratingFactors, InvariantEquation, InvariantSolutions, InvariantTransformation, Invariants, Laplace, Library, PDEplot, PolynomialSolutions, ReducedForm, SimilaritySolutions, SimilarityTransformation, Solve, SymmetryCommutator, SymmetryGauge, SymmetrySolutions, SymmetryTest, SymmetryTransformation, TWSolutions, ToJet, build, casesplit, charstrip, dchange, dcoeffs, declare, diff_table, difforder, dpolyform, dsubs, mapde, separability, splitstrip, splitsys, undeclare]

$Digits := 12$

$N := 4$

$\Phi_{r,\,0} := -r^2 + 1$

$\Phi_0 := -r^2 + 1$

Step 2:

```
for n from 0 to N do
F := P + υ·(diff(Φ[n], r$2) + 1/r·(diff(Φ[n], r))):
Φ[n + 1] := simplify(1/GAMMA(alpha)·int((r − s)^(alpha − 1)·F, s = 0 ..r), 'assume = nonnegative')
end do:
```

$$-r^2 + 1 + \frac{r^{\alpha}(P - 4\upsilon)}{\Gamma(\alpha + 1)}$$

$$0$$

$$0$$

$$0$$

$sol := sum(\Phi[k], k = 0 ..N + 1);$

$$sol := -r^2 + 1 + \frac{r^{\alpha}(P - 4\upsilon)}{\Gamma(\alpha + 1)}$$

Step 3:

$P := 5.0; \upsilon := 1;$
$T := evalf(sol);$
$sol1[0.2] := eval(T, [\alpha = 0.2]) : sol1[0.02] := eval(T, [\alpha = 0.02]) : sol1[0.002] := eval(T, [\alpha = 0.002]) :$

$P := 5.0$

$\upsilon := 1$

$$T := -1.\, r^2 + 1. + \frac{1.0\, r^{\alpha}}{\Gamma(\alpha + 1.)}$$

for i **from** 0 **by** 0.2 **to** 1 **do**
$\Phi[i] := evalf(eval(sol1[0.2], [r = i, t = 5]));$
end do;
for i **from** 0 **by** 0.2 **to** 1 **do**
$\Phi[i] := evalf(eval(sol1[0.02], [r = i, t = 5]));$
end do;
for i **from** 0 **by** 0.2 **to** 1 **do**
$\Phi[i] := evalf(eval(sol1[0.002], [r = i, t = 5]));$
end do;

$\Phi_0 := 1.$

$\Phi_{0.2} := 1.74937523160$

$\Phi_{0.4} := 1.74675403001$

$\Phi_{0.6} := 1.62334914897$

$\Phi_{0.8} := 1.40158686266$

$\Phi_{1.0} := 1.08912442106$

$\Phi_0 := 1.$

$\Phi_{0.2} := 1.93924807824$

$\Phi_{0.4} := 1.83291787200$

$\Phi_{0.6} := 1.64100247900$

$\Phi_{0.8} := 1.36677848897$

$\Phi_{1.0} := 1.01128165256$

$\Phi_0 := 1.$

$\Phi_{0.2} := 1.95793440512$

$\Phi_{0.4} := 1.83931879532$

$\Phi_{0.6} := 1.64012950180$

$\Phi_{0.8} := 1.36070510603$

$\Phi_{1.0} := 1.00115180748$

$p1 := plot3d(sol1[0.2], r = 0..1, t = -\pi..\pi, grid = [100, 100], color = \text{"blue"}) :$
$p2 := plot3d(sol1[0.4], r = 0..1, t = -\pi..\pi, grid = [100, 100], color = \text{"yellow"}) :$
$p3 := plot3d(sol1[0.6], r = 0..1, t = -\pi..\pi, grid = [100, 100], color = \text{"green"}) :$
$p4 := plot3d(sol1[0.8], r = 0..1, t = -\pi..\pi, grid = [100, 100], color = \text{"purple"}) :$
$p5 := plot3d(sol1[1.0], r = 0..1, t = -\pi..\pi, grid = [100, 100], color = \text{"red"}) :$
$plots{:-}display([p1, p2, p3, p4, p5]);$

$plot([\Phi[0.2], \Phi[0.4], \Phi[0.6], \Phi[0.8], \Phi[1.0]], r = -1..1, color = [blue, yellow, green, purple, red], axes = BOXED, title = \text{"Fractional order in a cylindrical time-fractional Navier-stoke equation } \Phi(r, 5)\text{"})$

$plot([\Phi[0.2], \Phi[0.02], \Phi[0.002]], r = -1..1, color = [red, gree, blue], axes = BOXED, title = \text{"Fractional order in a cylindrical time-fractional Navier-stoke equation } \Phi(r, 5)\text{"})$

$plot([\Phi[0.4], \Phi[0.04], \Phi[0.004]], r = -1..1, color = [red, gree, blue], axes = BOXED, title = \text{"Fractional order in a cylindrical time-fractional Navier-stoke equation } \Phi(r, 5)\text{"})$

$plot([\Phi[0.6], \Phi[0.06], \Phi[0.006]], r = -1..1, color = [red, gree, blue], axes = BOXED, title = \text{"Fractional order in a cylindrical time-fractional Navier-stoke equation } \Phi(r, 5)\text{"})$

$plot([\Phi[0.8], \Phi[0.08], \Phi[0.008]], r = -1..1, color = [red, gree, blue], axes = BOXED, title = \text{"Fractional order in a cylindrical time-fractional Navier-stoke equation } \Phi(r, 5)\text{"})$

$plot([\Phi[1.0], \Phi[0.10], \Phi[0.01]], r = -1..1, color = [red, gree, blue], axes = BOXED, title = \text{"Fractional order in a cylindrical time-fractional Navier-stoke equation } \Phi(r, 5)\text{"})$

Conflict of Interest

We the author of this paper declare that there is no conflict of interest.

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